Assignment in Mathematics for class: 12

1. The binary operation $*: R \times R \to R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$.

2. If the matrix $\begin{bmatrix} 6 & -x^2 \\ 2x - 15 & 10 \end{bmatrix}$ is symmetric; find the value of x.

3. Prove that: $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}\sqrt{x}$.

4. Without expanding at any stage, find the value of:
$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

5. Find the value of the constant 'K' so that the function $f(x)$ defined as:
$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, x \neq -1 \\ K, \quad x = -1 \end{cases}$$ is continuous at $x = -1$.

6. Show that the function $f(x)\colon \mathrm{N} \to \mathrm{N}$ defined by $f(x) = ax + b$; $a, b \in N$ is one-one but not onto.

7. Let $f\colon R_+ \to [-5, \infty)$, given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and hence find its inverse.

8. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that: $x + y + z = xyz$.

9. Using properties of determinants prove that:
$$\begin{vmatrix} \mathrm{a} & \mathrm{b} & \mathrm{b+c} \\ \mathrm{c} & \mathrm{a} & \mathrm{c+a} \\ \mathrm{b} & \mathrm{c} & \mathrm{a+b} \end{vmatrix} = (a+b+c)(a-c)^2.$$

10. Show that the function f defined as follows, is continuous at $x = 2$.
$$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$$

11. Using matrix method, solve the system of linear equations:
$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3.$$

12. Using elementary transformation, find the inverse of the matrix:
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

13. Prove that: $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$.

14. Without expanding the determinant, prove that: $\begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix} = x^3$.

15. Let $A = R - \{2\}$ and $B = R - \{1\}$. $f: A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that $f(x)$ is one-one and onto. Hence, find f^{-1}.

16. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

17. Using properties of determinants, prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$$

18. If $f: R \to R$ and $g: R \to R$ are defined respectively as $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find $fog(x)$ and $gof(x)$.

19. The binary operation $*$ defined on $Q - \{1\}$ is given by $a * b = a + b - ab$. Find the identity element.

20. Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - kM - I_2 = 0$.

21. Solve: $\sin^{-1}[\cos(\sin^{-1} x)] = \frac{\pi}{3}$.

22. Without expanding at any stage, find the value of:

$$\begin{vmatrix} 2 & x & y+z \\ 2 & y & z+x \\ 2 & z & x+y \end{vmatrix}$$

23. Let $f: N \to Y$ given by $f(x) = 4x^2 + 12x + 15$ and $y = range\ of\ f$. Show that f is invertible and hence find its inverse.

24. Solve the equation for x: $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x\,, x \neq 0$.

25. Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a & a \\ b & a+c & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

26. Given two matrices are A and B

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$$

Find AB and use this result to solve the following system of linear equations:

$$x - 2y + 3z = 6$$
$$x + 4y + z = 12$$
$$x - 3y + 2z = 1.$$

27. Using elementary transformation, find the inverse of the matrix:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

28. If A = $\begin{bmatrix} 5 & a \\ b & 0 \end{bmatrix}$ and A is a symmetric; show that a = b.

29. Solve for x: $\sin(2\tan^{-1}x) = 1$.

30. If a, b, c, are in A.P., find the value of:

$$\begin{bmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{bmatrix}.$$

31. $f(x) = \begin{cases} \frac{x^2-9}{x-3} \end{cases}$, is not defined at x = 3, What value should be assigned to f (3) for continuity of f(x) at x = 3.

32. If the function $f(x) = \sqrt{2x-3}$ is invertible then find its inverse. Hence prove that: $(f \circ f^{-1})(x) = x$.

33. If $\sec^{-1}x = \operatorname{cosec}^{-1}y$, show that : $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

34. Using the properties of determinants to solve the following for x:

$$\begin{bmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{bmatrix} = 0 \text{ and } x \neq 0$$

35. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

36. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

37. Using of properties of determinants prove that

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

38. Show that: $4\left(2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}\right) = \pi$

39. Solve the following equations by using matrix method :-

$$x + y = 2$$
$$x + 2y + 3z = 6$$
$$x - z = 0$$

40. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj (A) and verify that A(adj A) = (adj A).A

41. Solve the following equation:-

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)=\pi$$

42. Using the properties of determinants, prove that:-

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$$

43. Given the matrix A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, compute A^{-1}. Hence solve the equations:

x + 2y - 3z = - 4; 2x + 3y + 2z = 2 and 3x - 3y - 4z = 11.

44. Using the properties of determinants, show that:-

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$$

45. Solve for x:- $\cos(\sin^{-1}x) = \frac{1}{9}$.

46. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the system of equations x – y = 3; 2x + 3y + 4z =17 and y + 2z = 7 .

47. If $\begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix then find the value of x.

48. Find A^{-1}, where A = $\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$

Hence, solve the following system of linear equations:

4x + 2y + 3z = 2

x + y + z = 3

3x + y - 2z = 5

49. Solve for x: $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$.

50. If the matrix A = $\begin{bmatrix} 6 & x & 2 \\ -2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix, find the value of x

51. *If* $f(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, show that f(x) f(y) = f(x + y).

52. Using the properties of determinants, show that :

$$\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$$

53. Solve the following equation for x :

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x, \quad x > 0$$

54. Prove the following :

$$\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}.$$

55. If $f: R \to R$ and $g: R \to R$ are defined respectively as $f(x) = x^3$ and $g(x) = 2x^2 + 1$, find $fog(x)\ and\ gof(x)$.

56. Show that the function $f(x)$ in $A = R - \left\{\frac{8}{5}\right\}$ defined as $f(x) = \frac{8x+3}{5x-8}$ *i*s one-one and onto. Hence, find f^{-1}.

57. Show that binary operation $*$ defined on $R - \{-1\}$ is given by $a * b = a + b + ab$ is commutative and associative. Also find the identity element and prove that every element of given set is invertible.

58. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x^2 - y^2 - z^2 + 2yz\sqrt{1 - x^2} = 0$.

59. Solve for $x\ and\ y$ if $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} 2x \\ 3y \end{bmatrix} = 3\begin{bmatrix} 7 \\ -3 \end{bmatrix}$.

60. Using properties of determinants, prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).$$